

**S475/1**  
**SUBSID. MATHEMATICS**  
**Paper 1**  
**Jul. / Aug. 2018**

$2\frac{2}{3}$  hours



*"Together for Mathematics"*

**SECONDARY MATHEMATICS TEACHERS' ASSOCIATION**

**SMATA JOINT MOCK EXAMINATIONS 2018**

**Uganda Advanced Certificate of Education**

**SUBSIDIARY MATHEMATICS**

**Paper 1**

**2 hours 40 Minutes**

**INSTRUCTIONS TO CANDIDATES**

Answer all the **eight** questions in Section **A** and only **four** questions in Section **B**.

Any additional question(s) will **not** be marked.

Each question in section **A** carries **5** marks while each question in section **B** carries **15** marks.

All working **must** be shown clearly.

Begin each answer on a **fresh** sheet of paper.

Graph paper is provided.

Where necessary, take acceleration due to gravity,  $g = 9.8 \text{ ms}^{-2}$

Silent, non-programmable **scientific** calculators and mathematical tables with a list of formulae may be used.

## SECTION A: (40 MARKS)

Answer **all** questions in this Section.

- Given that  $\log_b m = X + Y$  and  $\log_b n = X - Y$   
Find the values of X and Y if  $\log_b \left(\frac{m^2}{n}\right) = 11$  and  $\log_b mn = 4$ . **(05 marks)**
- The age of the mother is twice her daughters' age. In five years' time, the product of their ages is 700. Find the current age of the mother  
**(05 marks)**
- The table below shows the marks of 8 students in the mid- term test and end of term test in subsidiary mathematics.

|                             |    |    |    |    |    |    |    |    |
|-----------------------------|----|----|----|----|----|----|----|----|
| <b>Mid-term test (x)</b>    | 99 | 77 | 50 | 67 | 77 | 81 | 96 | 67 |
| <b>End of term test (y)</b> | 99 | 55 | 35 | 60 | 75 | 70 | 99 | 50 |

Calculate the spearman's rank correlation coefficient. Comment on your results. **(05 marks)**

- The probability of the father being alive is 0.58 and the probability of his son being alive is 0.88  
What is the probability that
  - At least one of them will be alive
  - Both will die.**(05 marks)**

- Given that A  $\begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$  and B  $\begin{pmatrix} 2 & 4 \\ -2 & 0 \end{pmatrix}$   
Find i) AB  
ii) BA. Comment on your result **(05 marks)**

- The table below shows the frequency distribution of marks of 800 candidates who sat a national examination.

|                     |       |        |        |        |         |
|---------------------|-------|--------|--------|--------|---------|
| <b>Marks (%)</b>    | 0-<20 | 20-<40 | 40-<60 | 60-<80 | 80-<100 |
| <b>Frequency, f</b> | 140   | 160    | 180    | 250    | 70      |

- Calculate the class width
- State the modal class and median class. Hence, calculate the median mark. **(05 marks)**

7. Given that  $\frac{dm}{dt} = -m$ . When  $t = 0$ ,  $m = 64\text{g}$ . Find  $m$ , when  $t = 2$  hours.  
(05 marks)
8. In a rectangle ABCD,  $AB = 4\text{cm}$  and  $BC = 3\text{cm}$ . forces of magnitude 3N, 10 N, 4N, 6N and 5N act in the directions of the letters AB, BC, CD, DA and AC respectively. Taking AB as a horizontal, find the magnitude of the resultant forces.  
(05 marks)

### SECTION B : ( 60 MARKS)

Answer any **four** questions from this section

9. Given below is the probability distribution of the number of compact discs (CDs) sold  $P(X = 3) = P(X = 5) = k$  ;  $P(X = 1) = P(X = 4) = 3 P(X = 3)$ ;  
 $P(X = 2) = 2 P(X = 5)$   
Determine the:
- Value of  $k$ .
  - Probability that at least 3 CDs are sold
  - Expectation of CDs sold
  - Standard deviation.
- (15 marks)

10. A farmer recorded his annual output of beans for 8 years, as shown in the table

| Year               | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|--------------------|------|------|------|------|------|------|------|
| Output (in tonnes) | $x$  | 28   | 30   | 29   | 26   | 29   | $y$  |

- Calculate the 3-year moving averages
  - On the same axes represent the annual output and the 3-year moving averages.
  - Use your graph to estimate the values of  $x$  and  $y$ .
- (15 marks)
11. a) The polynomial  $f(x) = px^3 + qx^2 - 4$  is divisible by  $x + 2$  and has a remainder of 15 when divided by  $x - 1$ . Find the values of  $p$  and  $q$ .
- b) The roots of the quadratic equation  $2x^2 + 5x - 12 = 0$  are  $\alpha$  and  $\beta$ . Find the quadratic equation whose roots are  $\alpha^2\beta$  and  $\beta^2\alpha$ . Show that the roots of this equation are real if and only if  $b^2 \geq 4ac$ .  
(15 marks)

12. The rate of decay of a substance is directly proportional to its mass, M. Initially the original mass was 96g. When  $t = 4$  hours,  $M = 48$ g. Find the mass when  $t = 16$  hours. **(15 marks)**

13. The table below shows part of a shopping list of Vicky

|                  | <b>June</b> |          | <b>July</b> |          |
|------------------|-------------|----------|-------------|----------|
| <b>ITEM</b>      | Price       | Quantity | Price       | Quantity |
| Beans ( kg)      | 2000        | 2        | 1800        | 2        |
| Maize flour(kg)  | 1500        | 5        | 1700        | 4        |
| Wheat flour(kg)  | 3000        | 1        | 2800        | 1        |
| Peas(kg)         | 3000        | 2        | 3400        | 1        |
| Millet flour(kg) | 2600        | 2        | 3000        | 3        |

Taking July as the base, calculate;

- (a) price relative of each commodity.
- (b) weighted price index, of June hence, comment on your answer. **(15 marks)**
14. A body of mass 2kg rests on a rough plane inclined at an angle of  $30^\circ$  to the horizontal. A 2kg mass is attached to a string which goes up the greatest slope, passes over a fixed pulley at the edge, and attached to a 3kg mass which hangs freely. If the system is released from rest, a 3kg mass covers a distance of 0.5m and hits the ground with a velocity of  $2\text{ms}^{-1}$ . (Coefficient of friction = 0.2, use  $g = 10\text{ms}^{-2}$ ). Calculate;
- (a) Acceleration of the system
- (b) Tension in the string
- (c) Reaction at the pulley. **(15 marks)**

**END**

## SMATA MARKING GUIDE.

|    |                                                     |      |                |                              |
|----|-----------------------------------------------------|------|----------------|------------------------------|
| 1. | $\log_6 m^2 - \log_6 n = 11$                        | ← or | M <sub>1</sub> | mobile. (Any                 |
|    | $2\log_6 m - \log_6 n = 11$                         |      |                |                              |
|    | $2(x+y) - (x-y) = 11$                               |      |                |                              |
|    | $x + 3y = 11 - (1)$                                 | ← or | B <sub>1</sub> | mobile.                      |
|    | $\log_6 m \times \log_6 n = 4$                      | ←    |                |                              |
|    | $(x+y)(x-y) = 4$                                    |      |                |                              |
|    | $x^2 - y^2 = 4 - (2)$                               | ←    |                |                              |
|    | $x = 11 - 3y$                                       |      | M <sub>1</sub> |                              |
|    | $(11 - 3y)^2 - y^2 = 4$                             |      | M <sub>2</sub> | correct substitution         |
|    | $121 - 66y + 9y^2 - y^2 = 4$                        |      |                |                              |
|    | $8y^2 - 66y + 117 = 0$                              |      |                |                              |
|    | Using a calculator                                  |      |                |                              |
|    | Either $y = 5.671$ or $y = 2.579$                   |      | A <sub>1</sub> | Both answers<br>Accept 1 dp. |
|    | $x = 11 - 3 \times 5.671$ $x = 11 - 3 \times 2.579$ |      |                |                              |
|    | $x = -6.013$ $x = 3.263$                            |      | A <sub>1</sub> | Both answers<br>Accept 1 dp. |
|    |                                                     |      | 5mks           |                              |
| 2. | daughter   Mother                                   |      |                |                              |
|    | $x$ $2x$                                            |      |                |                              |
|    | $(x+5)$ $(2x+5) = 700$                              |      | B <sub>1</sub> |                              |
|    | $(x+5)(2x+5) = 700$                                 |      |                |                              |
|    | $2x^2 + 5x + 10x + 25 = 700$                        |      | M <sub>1</sub> | Attempting to<br>solve       |
|    | $2x^2 + 15x - 675 = 0$                              |      |                |                              |
|    | Using a calculator                                  |      |                |                              |
|    | $x = 15$ or $x = -22.5$                             |      | A <sub>1</sub> | Both                         |
|    | $\therefore x = 15$                                 |      | A <sub>1</sub> | correct ans                  |
|    | Current age of the mother = $2 \times 15 + 5$       |      | M <sub>1</sub> |                              |
|    | $= 30 \text{ years}$                                |      | A <sub>1</sub> |                              |
|    |                                                     |      | 5mks           |                              |

| 3. | X  | Y  | $R_x$ | $R_y$ | d    | $d^2$             |       |                |
|----|----|----|-------|-------|------|-------------------|-------|----------------|
|    | 99 | 99 | 1     | 1.5   | -0.5 | 0.25              | $B_1$ | Both R.        |
|    | 77 | 55 | 4.5   | 6     | -1.5 | 2.25              |       | correct        |
|    | 50 | 35 | 8     | 8     | 0    | 0                 | $B_1$ | $d^2$ correct  |
|    | 67 | 60 | 6.5   | 5     | 1.5  | 2.25              |       |                |
|    | 77 | 75 | 4.5   | 3     | 1.5  | 2.25              |       |                |
|    | 81 | 70 | 3     | 4     | -1   | 1                 |       |                |
|    | 96 | 99 | 2     | 1.5   | 0.5  | 0.25              |       |                |
|    | 67 | 50 | 6.5   | 7     | -0.5 | 0.25              |       |                |
|    |    |    |       | $B_1$ |      | $\Sigma d^2 = 85$ |       | $B_1$<br>$B_2$ |

Correct substitution  $M$

(Accept 1 dp)  $\rightarrow A$

underlined must be seen irrespective of the copy.

$$= 0.0504 \quad \text{A7}$$

19

5. i)  $AB = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ -2 & 0 \end{pmatrix}$   
 $= \begin{pmatrix} 6 & 4 \\ -4 & 4 \end{pmatrix}$

M1 correct substn

A1 correct ans.

ii)  $BA = \begin{pmatrix} 2 & 4 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$   
 $= \begin{pmatrix} 6 & -8 \\ -2 & 4 \end{pmatrix}$

M1

A1

$AB \neq BA$

B1 accept in either words form..

6i) class width =  $80 - 40$   
 $= 20$

B1

ii) modal class =  $60 - 80$

B1

| marks  | f   | c.f |
|--------|-----|-----|
| 0-20   | 140 | 140 |
| 20-40  | 160 | 300 |
| 40-60  | 180 | 480 |
| 60-80  | 250 | 730 |
| 80-100 | 70  | 800 |

B1 All correct values of c.f.

$\frac{N}{2} = \frac{800}{2} = 400$

Median class =  $40 - 60$

B1

Median =  $40 + \left( \frac{400 - 300}{180} \right) 20$   
 $= 51.111\%$

B1

A1 Accept 1 dp without units

(5 marks)

7.  $\frac{dm}{dt} = -m$

$$\int \frac{dm}{m} = \int -dt$$

$$\ln m = -t + C$$

$$\ln 64 = -0 + C$$

$$C = \ln 64$$

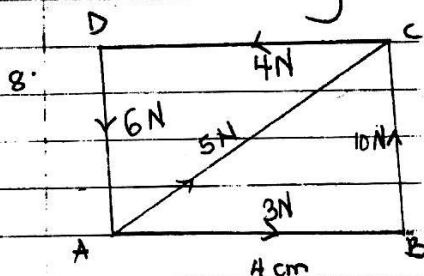
$$\ln m = -t + \ln 64$$

$$\ln m = -2 + \ln 64$$

$$\ln m = 2.15888$$

$$m = e^{2.15888}$$

$$m = 8.661 \text{ g}$$



$$AC = \sqrt{4^2 + 3^2} = 5$$

$$F = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 10 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \end{pmatrix} + \begin{pmatrix} 5 \times \frac{4}{5} \\ 5 \times \frac{3}{5} \end{pmatrix}$$

$$F = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \text{ N}$$

$$F = \sqrt{3^2 + 7^2}$$

$$= 7.6158 \text{ N}$$

M<sub>1</sub> - separation of variable  
- integral signs &  
dm & dt seen

M<sub>1</sub> with a constant

B<sub>1</sub> value of the constant

M<sub>1</sub> correct substitution

A<sub>1</sub> correct output  
5mks accept 2 d.p.s. with or  
without units

B<sub>1</sub> labelled diagram with  
all forces indicated

B<sub>1</sub> Resultant ~~at~~ horizontal

B<sub>1</sub> net vertical

M<sub>1</sub>

A<sub>1</sub> (Accept 2 d.p.s)

5mks

follow the candidate

P5

EPZ

9. P(x=x)

| X | P(x=x) | P(x=x) | x P(x=x) | X <sup>2</sup> | X <sup>2</sup> P(x=x) |
|---|--------|--------|----------|----------------|-----------------------|
| 1 | 3K     | 3/9    | 3/9      | 1              | 3/9                   |
| 2 | 2K     | 2/9    | 4/9      | 4              | 8/9                   |
| 3 | K      | 1/9    | 3/9      | 9              | 9/9                   |
| 4 | 2K     | 3/9    | 12/9     | 16             | 48/9                  |
| 5 | K      | 1/9    | 5/9      | 25             | 25/9                  |
|   | 9K     |        | 27/9     |                | 93/9                  |

$$9K = 1$$

$$K = 1/9$$

All correct  
P(x=x)  
with K =  
Σ P(x=x)  
All correct  
Σ x P(x=x)  
Σ x P(x=x)  
Σ x<sup>2</sup> P(x=x) all  
correct  
Σ x<sup>2</sup> P(x=x)

b. P(At least 3 CDs) =  $\frac{1}{9} + \frac{3}{9} + \frac{1}{9}$   
=  $\frac{5}{9}$  or

c. E(X) =  $\frac{27}{9}$

= 3

d. S.d(x)

$$S.d = \sqrt{\frac{93}{9} - 3^2}$$

$$= \sqrt{\frac{4}{3} \cdot \frac{12}{3}}$$

= 1.333

substn for K  
Σ P(x=x) = 1  
Equating to 1.  
value of K in  
fractions  
accept idp's

correct ans

correct substn  
under a root

accept idp  
15 Mks

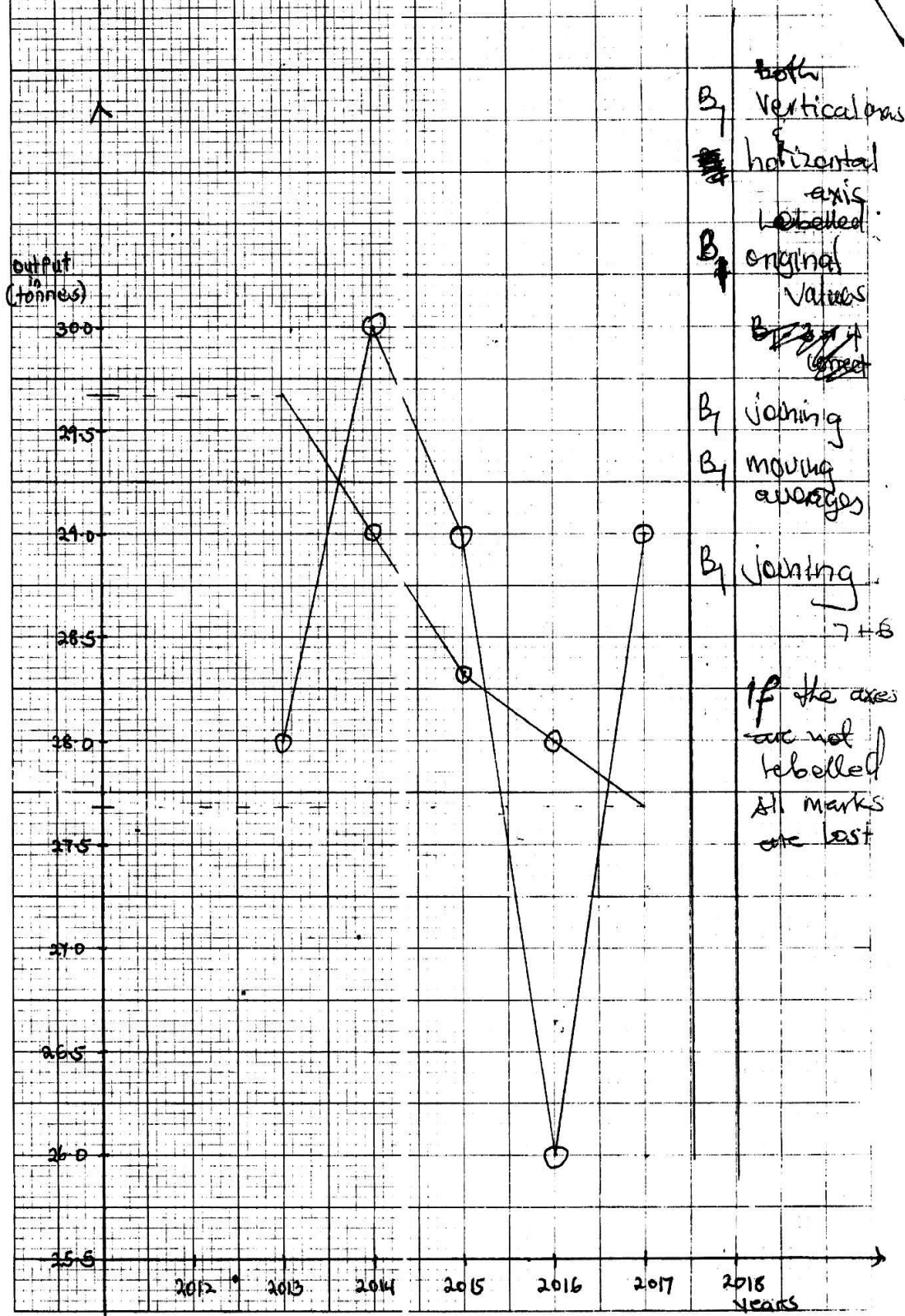
| 10. Year | Out Put (tonnes) | 3-Point moving totals | 3-Point moving averages |
|----------|------------------|-----------------------|-------------------------|
| 2012     | X                |                       |                         |
| 2013     | 28               | x + 58                | $\frac{x+58}{3}$        |
| 2014     | 30               | 87                    | 29                      |
| 2015     | 29               | 85                    | 28.333                  |
| 2016     | 26               | 81                    | 28                      |
| 2017     | 29               | 7+55                  | $\frac{7+55}{3}$        |
| 2018     | 7                |                       |                         |

moving totals  
moving averages

|     |                                                                                 |   |                                |
|-----|---------------------------------------------------------------------------------|---|--------------------------------|
| 12. | $\frac{dM}{dt} \propto M$                                                       | B |                                |
|     | $\frac{dM}{dt} = -kM$                                                           | B | is a -ve                       |
|     | $dM = -kdt$                                                                     | M | separatn of variables          |
|     | $M$                                                                             |   |                                |
|     | $\ln M = -kt + C$                                                               | M | Correct integratn w a constant |
|     | $t=0, M=96$ or $\ln 96 = -k \times 0 + C$                                       | M |                                |
|     | $\ln M = -kt + \ln 96$ or $\ln M = -kt + \ln 96$                                | B | Substn for a constant          |
|     | $k = \frac{-1}{t} \ln \left( \frac{M}{96} \right)$                              |   |                                |
|     | $t=4, M=48$                                                                     |   |                                |
|     | $k = \frac{-1}{4} \ln \left( \frac{48}{96} \right)$                             | M | Substn                         |
|     | $k = \frac{-1}{4} \ln \left( \frac{1}{2} \right)$                               |   |                                |
|     | $\ln \left( \frac{M}{96} \right) = \frac{t}{4} \ln \left( \frac{1}{2} \right)$  | B | value of k                     |
|     | $\ln \left( \frac{M}{96} \right) = \frac{16}{4} \ln \left( \frac{1}{2} \right)$ | M | Eqn                            |
|     | $\ln \left( \frac{M}{96} \right) = 4 \ln \left( \frac{1}{2} \right)$            | M |                                |
|     | $\frac{M}{96} = e^{4 \ln \frac{1}{2}}$                                          |   |                                |
|     | $M = 96 \cdot e^{4 \ln \frac{1}{2}}$                                            | M |                                |
|     | $M = 6g$                                                                        | A | Accept 0dp                     |
|     | $M = 5.9987$                                                                    |   |                                |
|     |                                                                                 |   | 15 MARKS                       |

|                                                                                                                                                                                                      |                                                                                                                                                     |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| 139. Beans: $\frac{2000}{1800} = 1.111$                                                                                                                                                              | M <sub>1</sub> division                                                                                                                             |
| M. Flour: $\frac{1500}{1700} = 0.882$                                                                                                                                                                | B <sub>1</sub>                                                                                                                                      |
| W. Flour: $\frac{3000}{2800} = 1.071$                                                                                                                                                                | B <sub>1</sub>                                                                                                                                      |
| Peas : $\frac{3000}{3400} = 0.882$                                                                                                                                                                   | B <sub>1</sub>                                                                                                                                      |
| Millet flour: $\frac{2600}{3000} = 0.867$                                                                                                                                                            | B <sub>1</sub>                                                                                                                                      |
| b. W.P.I = $\frac{(2000 \times 2) + (1500 \times 5) + (3000 \times 1) + (3000 \times 2) + (2600 \times 2)}{(1800 \times 2) + (1700 \times 4) + (2800 \times 1) + (3400 \times 1) + (3000 \times 3)}$ | M <sub>1</sub> Multiplication sum<br>M <sub>1</sub> Denominator sum<br>M <sub>1</sub> Numerator for multiplication<br>M <sub>1</sub> = $\times 100$ |
| = $\frac{4000 + 7500 + 3000 + 6000 + 5200}{3600 + 6800 + 2800 + 3400 + 9000} \times 100$                                                                                                             |                                                                                                                                                     |
| = $\frac{25700}{25600} \times 100$                                                                                                                                                                   | B <sub>1</sub> output denominator                                                                                                                   |
| = 100.391                                                                                                                                                                                            | B <sub>1</sub> output denominator                                                                                                                   |
| The prices were high in June by 0.391% compared to July                                                                                                                                              | A <sub>1</sub>                                                                                                                                      |
| In a table $\left\{ \begin{array}{l} \text{Numerator } M_1 \\ \text{Denominator } M_1 \end{array} \right.$ All correct pdt in a column                                                               | 15 MKS                                                                                                                                              |
| $\left\{ \begin{array}{l} M_1, B_1 \text{ correct sum} \end{array} \right.$                                                                                                                          |                                                                                                                                                     |
| $\left\{ \begin{array}{l} \text{Denominator } M_1 \\ M_1, B_1 \text{ correct sum} \end{array} \right.$                                                                                               |                                                                                                                                                     |
| M <sub>1</sub> division and $\times 100$                                                                                                                                                             |                                                                                                                                                     |
| A <sub>1</sub> correct output                                                                                                                                                                        |                                                                                                                                                     |
| B <sub>1</sub> comment                                                                                                                                                                               |                                                                                                                                                     |

Q5 P6



|                 |                                             |                       |                                       |
|-----------------|---------------------------------------------|-----------------------|---------------------------------------|
| 10c:            | $\frac{x+58}{3} = 29.675$                   | Accept (29.65, 29.75) | M <sub>1</sub>                        |
|                 | $x = 31.025$ tonnes                         | (30.8 31.25)          | A <sub>1</sub>                        |
|                 | $\frac{y+55}{3} = 27.675$                   | Accept (27.65 27.70)  | M <sub>1</sub>                        |
|                 | $y = 28.025$ tonnes                         | (27.95 28.1)          | A <sub>1</sub>                        |
| <b>15 MARKS</b> |                                             |                       |                                       |
| 11a:            | $f(-2) = -8p + 4q - 4 = 0$                  |                       | M <sub>1</sub> subtn & equate to zero |
|                 | $-8p + 4q = 4$                              |                       |                                       |
|                 | $-2p + q = 1 \quad \text{--- (1)}$          |                       | B <sub>1</sub> correct eqn.           |
|                 | $f(1) = p + q - 4 = 15$                     |                       | M <sub>1</sub> subtn & equate to 15   |
|                 | $p + q = 19 \quad \text{--- (2)}$           |                       | B <sub>1</sub>                        |
|                 | ② - ①                                       |                       |                                       |
|                 | $(p+q) - (-2p+q) = 19-1$                    |                       | M <sub>1</sub> Attempting to solve    |
|                 | $3p = 18$                                   |                       |                                       |
|                 | $p = 6$                                     |                       | A <sub>1</sub>                        |
|                 | $q = 19-6$                                  |                       |                                       |
|                 | $q = 13$                                    |                       | A <sub>1</sub>                        |
| b.              | $x^2 + 5x - 6 = 0$                          |                       |                                       |
|                 | $a+p = -\frac{5}{2} \quad qp = -6$          |                       | B <sub>7</sub> Both pdt & sum         |
|                 | $a^2p + p^2a = a^2p^2(a+p)$                 |                       | M <sub>1</sub>                        |
|                 | $= (-6)^2 \times \frac{5}{2}$               |                       | M <sub>1</sub>                        |
|                 | $= -90$                                     |                       | B <sub>1</sub>                        |
|                 | $(a^2p) / (p^2a) = a^3p^3$                  |                       |                                       |
|                 | $= (-6)^3$                                  |                       |                                       |
|                 | $= -216$                                    |                       | B <sub>7</sub>                        |
|                 | $x^2 + 90x - 216 = 0$ is the equation.      |                       | A <sub>1</sub>                        |
|                 | $b^2 - 4ac = 90^2 - 4 \times 1 \times -216$ |                       | M <sub>1</sub>                        |
|                 | $= 8100 + 864 > 0$                          |                       | A <sub>1</sub>                        |
|                 | Hence it will have a real root              |                       |                                       |
| <b>15 MARKS</b> |                                             |                       |                                       |

5305